Trabajo Práctico N° 2
Principio de los Trabajos Virtuales

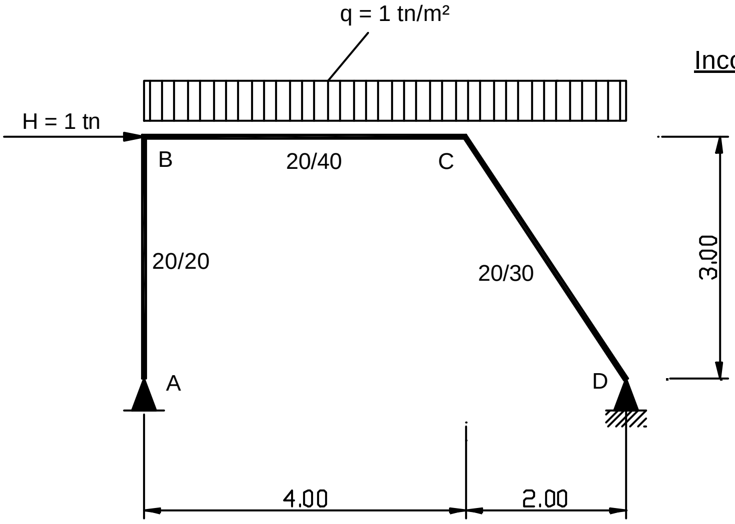
Calcular la rotación del punto A “F_A”

Datos:

E = 2100000 tn/m^2

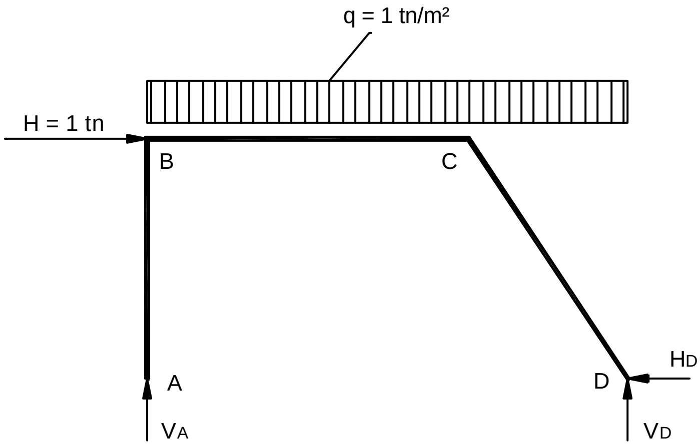
Incognitas:

Φ_A= ?



Estado 0

Diagrama de Cuerpo Libre para el Estado 0



Cálculo de Reacciones:

$$\sum M_A = 0 \Rightarrow -V_D \times 6\,m + \frac{q \times (6m)^2}{2} + H \times 3m = 0 \Rightarrow V_D = \frac{1tn/m \times (6m)^2}{2 \times 6m} + \frac{1tn \times 3m}{6m} \Rightarrow \left[\mathbf{V_D = 3,50\,tn}\right]$$

$$\sum M_D = 0 \Rightarrow V_A \times 6\,m - \frac{q \times (6m)^2}{2} + H \times 3m = 0 \Rightarrow V_A = \frac{1tn/m \times (6m)^2}{2 \times 6m} - \frac{1tn \times 3m}{6m} \Rightarrow \left[\mathbf{V_A = 2,50\,tn}\right]$$

$$\sum F_x = 0 \Rightarrow H - H_D = 0 \Rightarrow \left[\mathbf{H_D = H = 1,00\,tn}\right]$$

Cálculo de los Momentos

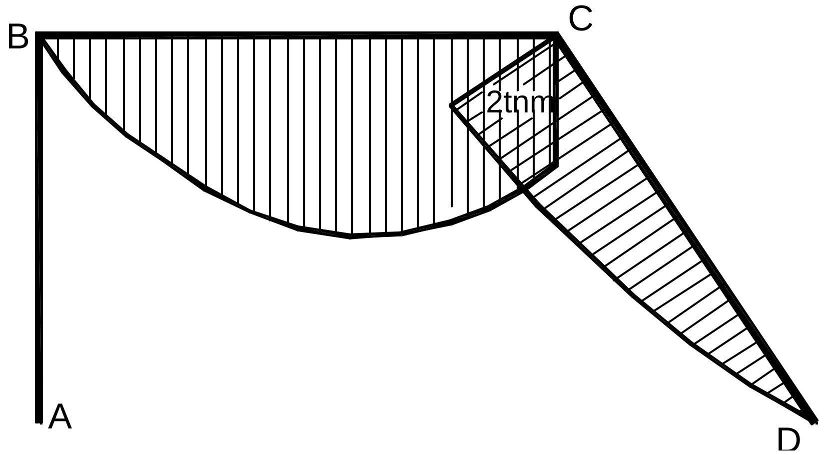
$M_A^I = \mathbf{0}$

$M_B^I = M_B^D = \mathbf{0}$

$M_C^I = M_C^D = H_D \times 3m - V_D \times 2m + \frac{q \times (2m)^2}{2} = -\mathbf{2\ tnm}$

$M_D = \mathbf{0}$

Diagrama de Momento Para el **Estado 0**



ESTADO 1

Estado de Carga

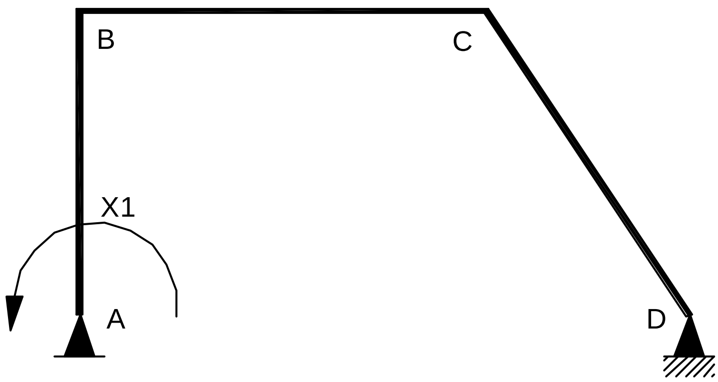
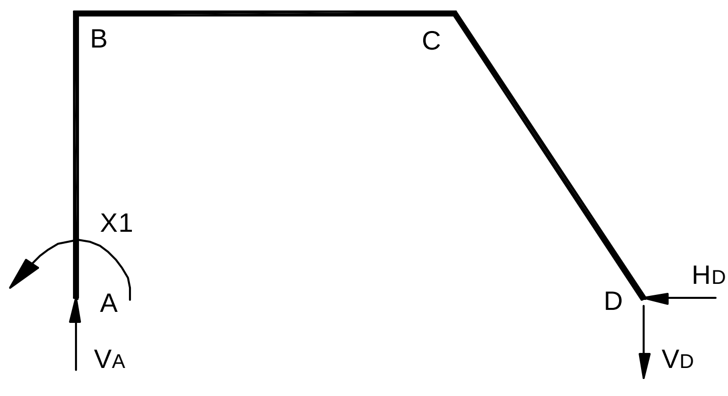


Diagrama de Cuerpo Libre



Cálculo de Reacciones

$$\sum M_A = 0 \Rightarrow V_D \times 6\,m - X_1 = 0 \Rightarrow V_D = \frac{X_1}{6m} = \frac{1tnm}{6m} \Rightarrow \left[\mathbf{V_D = 0,17\,tn}\right]$$

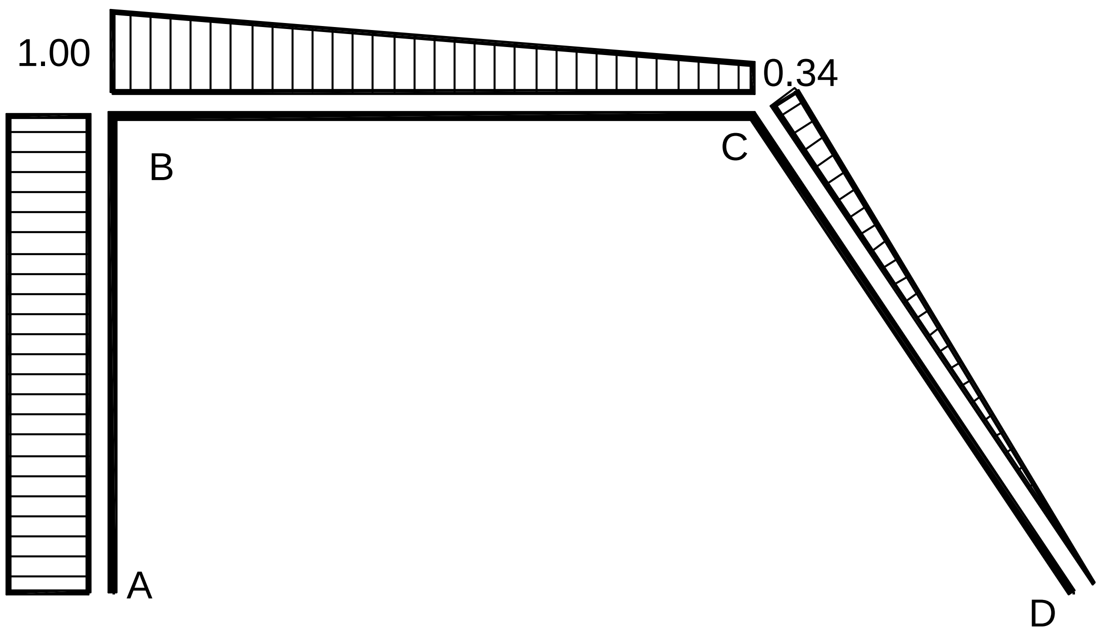
$$\sum M_D = 0 \Rightarrow V_A \times 6\,m - X_1 = 0 \Rightarrow V_A = \frac{1tnm}{6m} \Rightarrow \left[\mathbf{V_A = 0.17\,tn}\right]$$

$$\sum F_x = 0 \Rightarrow \left[\mathbf{H_D = 0}\right]$$

Cálculo de los Momentos

$$M_A^I = \mathbf{1tnm} \quad \leftarrow$$
$$M_B^I = M_B^D = \mathbf{1tnm} \uparrow$$
$$M_C^I = M_C^D = V_D \times 2m = \mathbf{0.34tnm}$$
$$M_D = \mathbf{0}$$

Diagrama de Momento Para el **Estado 1**



$$Te = \text{-}Ti = X_1 \times \Phi_A = \int_0^l \frac{M \times M^*}{E \times I} dx$$

$$\Phi_A = \frac{1}{X_1} \times \left[\int_A^B \frac{M \times M^*}{E \times I} dx + \int_B^C \frac{M \times M^*}{E \times I} dx + \int_C^D \frac{M \times M^*}{E \times I} dx \right]$$

Cálculo de las Inercias

$$I_{AB} = \frac{0.20\,m \times (0.20\,m)^3}{12} = \mathbf{0.00013m^4}$$

$$I_{BC} = \frac{0.20\,m \times (0.40\,m)^3}{12} = \mathbf{0.00107m^4}$$

$$I_{CD} = \frac{0.20\,m \times (0.30\,m)^3}{12} = \mathbf{0.00045m^4}$$

$$\text{Adopto como } I_0 = I_{BC} = \mathbf{0.00107m^4}$$

Cálculo de los Coeficientes "a_{ik}"

$$\alpha = \frac{I_{ik}}{I_0}$$

$$\Rightarrow \alpha_{AB} = \frac{I_{AB}}{I_0} = \frac{0.00013\ m^4}{0.00107\ m^4} \Rightarrow [\mathbf{a_{AB} = 0.121}]$$

$$\Rightarrow \alpha_{BC} = \frac{I_{BC}}{I_0} = \frac{0.00107\ m^4}{0.00107\ m^4} \Rightarrow [\mathbf{a_{BC} = 1}]$$

$$\Rightarrow \alpha_{CD} = \frac{I_{CD}}{I_0} = \frac{0.00045\ m^4}{0.00107\ m^4} \Rightarrow [\mathbf{a_{CD} = 0.422}]$$

$$\Phi_A = \frac{1}{X_1} \times \left[\int_A^B \frac{M \times M^*}{E \times \alpha_{AB} \times I_0} dx + \int_B^C \frac{M \times M^*}{E \times \alpha_{BC} \times I_0} dx + \int_C^D \frac{M \times M^*}{E \times \alpha_{CD} \times I_0} dx \right] =$$

$$= \frac{1}{X_1 \times I_0 \times E} \times \left[\int_A^B \frac{M \times M^*}{\alpha_{AB}} dx + \int_B^C \frac{M \times M^*}{\alpha_{BC}} dx + \int_C^D \frac{M \times M^*}{\alpha_{CD}} dx \right] \qquad (\mathbf{1})$$

Cálculo de las Integrales por Tramo

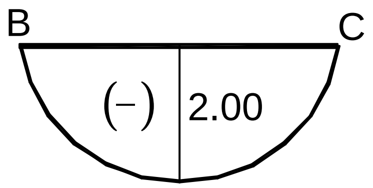
Tramo AB: l_{AB} = 3,00 m ; α_{AB} = 0.121

Nota: Como el Momento Correspondiente al **Estado 0**, para este tramo es igual a **cero,** el valor de la integral para dicho tramo también valdrá **cero-**

$$\int_A^B \frac{M \times M^* 0}{\alpha_{AB}} dx = 0$$

Tramo BC: l_{BC} = 4,00m ; α_{AB} = 1

Diagrama de Momento p/el **Estado 0**



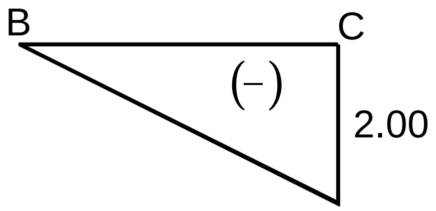
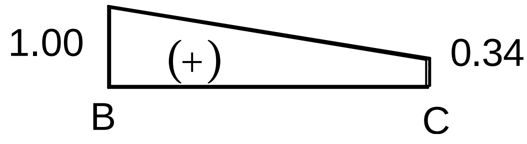


Diagrama de Momento p/el **Estado 1**



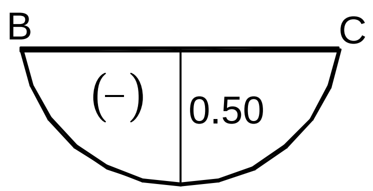
Con Estos Diagramas Entramos a Tabla y Obtenemos los siguientes datos:

$$\int_B^C \frac{M\times M*0}{\alpha_{BC}}dx = \frac{4m}{1}\times\left[\left(\frac{1}{3}\times(1tnm+0.34tnm)\right)\times(-2tnm)+\left(\frac{1}{6}\times(1tnm+2\times 0.34tnm)\times(-2tnm)\right)\right]\Rightarrow$$

$$\Rightarrow\left\langle \int_B^C \frac{\mathbf{M\times M*}}{\mathbf{a_{BC}}}\mathbf{dx} = \mathbf{-10.30\ tn^2m^3}\right\rangle \quad \mathbf{(2)}$$

Tramo CD: l_{CD} = 3.61m ; α_{AB} = 0.421

Diagrama de Momento p/el **Estado 0**



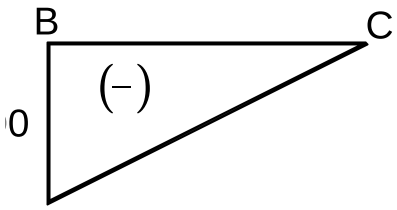
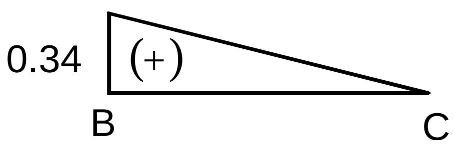


Diagrama de Momento p/el **Estado 1**



$$\int_C^D \frac{M \times M^*}{\alpha_{CD}} dx = \frac{3.61m}{1} \times \left[\left(\frac{1}{3} \times 0.34tnm \times (-0.5tnm) \right) + \left(\frac{1}{3} \times 0.34tnm \times (-2tnm) \right) \right] \Rightarrow$$

$$\Rightarrow \left\langle \int_C^D \frac{\mathbf{M \times M^*}}{\mathbf{a_{CD}}} \mathbf{dx} = -2\mathbf{.42\ tn^2 m^3} \right\rangle \qquad \mathbf{(3)}$$

Reemplazando (2) y (3) en (1) y los otros datos “E” y “I_o”, tenemos:

$$\Phi_A = \frac{1}{X_1 \times I_0 \times E} \times \left[\int_A^B \frac{M \times M^*}{\alpha_{AB}} dx + \int_B^C \frac{M \times M^*}{\alpha_{BC}} dx + \int_C^D \frac{M \times M^*}{\alpha_{CD}} dx \right] =$$

$$= \frac{1}{1tnm \times 0.00107m^4 \times 2100000\ tn/m^2} \times \left(0 - 10.30tn^2m^3 - 2.42\,tn^2m^3 \right) \Rightarrow$$

$$\Rightarrow \left\| \mathbf{F_A = -0.0057 = -0°0'20.59''} \right\|$$